Double-beta-decay suppression with three generations of Majorana neutrinos

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Rate suppression of double-beta decay with three generations of Majorana neutrinos is analyzed. It is shown that the physically attractive possibility of three equal masses and a relative *CP* eigenvalue of the first different from the other two can lead to complete cancellation in double-beta decay. An explanation in terms of pseudo-Dirac neutrinos is offered, and the process $\mu^- \to e^+$ is also examined.

The possible Majorana and Dirac properties of massive neutrinos have been the subject of several studies in the last few years. The fact that two massive Majorana neutrinos can mix to form a Dirac neutrino has been used to explain a possible suppression of neutrinoless double-beta decay.[1,2] The motivation here was to make the neutrino mass constraint obtained from double-beta decay[3] consistent with the mass value obtained in tritium beta decay.[4] The correct interpretation of the two equal-mass neutrinos and their right-handed partners leads to a mass matrix of the Dirac type, in such a way that the lepton number conserved by the mass matrix is the same as that conserved by the weak interaction. However, Wolfenstein[5] recognized that such an arrangement of Majorana states need not conserve the lepton number of the weak interaction. In that case the states would be related to the weak eigenstates by an orthogonal transformation, and he called them pseudo-Dirac neutrinos.

In the case of two generations, double-beta decay would still proceed with pseudo-Dirac neutrinos. That occurs because higher-order weak-interaction corrections to the mass will split the two Majorana particles and they will end up with a mass difference. A three-generation study was also carried out,[6] but the neutrino states were chosen in such a way that full suppression could not occur.

In this paper we study possible rate suppression with three generations of Majorana neutrinos. An attractive possibility that we analyze is that of three equal-mass states. In that case, if the relative *CP* eigenvalue of the first is opposite from those of the other two, the suppression of double-beta decay is complete. We find that the mixing with the third neutrino generates a mass splitting in the other two which turns them into a pseudo-Dirac neutrino. The pseudo-Dirac neutrino would allow double-beta decay, but its contribution is exactly canceled by the remaining Majorana neutrino. Other processes involve different parts of the mixing matrix and different effects occur. For example, we find that for $\mu^- \to e^+$ conversion full cancellation is obtained for three neutrinos of equal *CP* eigenvalues.

If we assume that neutrinos are Majorana particles, the mass term in the Lagrangian can be written in the form

$$-L_m = (\nu_e^T \nu_\mu^T \nu_\tau^T) CM \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} , \tag{1}$$

where ν_e, ν_μ, ν_τ are the weak eigenstates, M is a 3×3 symmetric matrix, and C is the charge-conjugation matrix. M can be diagonalized with a unitary matrix U and, for the case of three generations, the lepton phases can be defined in such a way that U is left with three *CP*-violating phases. Then the mass eigenvalues are obtained from

$$U^T M U = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} , \tag{2}$$

where m_1, m_2, m_3 are real and positive. The matrix which accomplishes this transformation is[7–9]

$$U = \begin{pmatrix} C_1 & -S_1C_3e^{i\xi} & -S_1S_3e^{i\eta} \\ S_1C_2e^{-i\xi} & (C_1C_2C_3 - S_2S_3e^{i\delta}) & (C_1C_2S_3 + S_2C_3e^{i\delta})e^{i(\eta-\xi)} \\ S_1S_2e^{-i\eta} & (C_1S_2C_3 + C_2S_3e^{i\delta})e^{-i(\eta-\xi)} & (C_1S_2S_3 - C_2C_3e^{i\delta}) \end{pmatrix} , \tag{3}$$

where $C_i = \cos\theta_i$, $S_i = \sin\theta_i$, θ_i are three mixing angles and ξ, η, δ are three *CP*-violating phases. The mass eigenstates are then

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = U^{-1} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L . \tag{4}$$

With a definition of the Majorana fields

$$\chi_i = \nu_{iL} + \nu_{iR}^c , \tag{5}$$

such that $\chi_i^c = \chi_i$, the amplitude for double-beta decay mediated by massive Majorana neutrinos[9] is proportional to

$$\sum_j m_j U_{1j}{}^2 + m_1 C_1{}^2 + m_2 S_1{}^2 C_3{}^2 e^{2i\xi} + m_3 S_3{}^2 e^{2i\eta} . \tag{6}$$

There is a second approach in dealing with the *CP*-violating phases. It is possible to redefine the neutrino fields:

$$\chi_2' = \chi_2 e^{i\xi} , \quad \chi_3' = \chi_3 e^{i\eta} . \tag{7}$$

With a redefinition of the leptons $\mu' = \mu e^{i\xi}$, $\tau' = \tau e^{i\eta}$ we obtain a mixing matrix with only one phase left in it. That is,

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$$J^\alpha=(\overline{e}\,\overline{\mu}'\tau')_L\gamma^\alpha\begin{pmatrix} C_1 & -S_1C_3 & -S_1S_3\\ S_1C_2 & C_1C_2C_3-S_2S_3e^{i\delta} & C_1C_2S_3+S_2C_3e^{i\delta}\\ S_1S_2 & C_1S_2C_3+C_2S_3e^{i\delta} & C_1S_2S_3-C_2C_3e^{i\delta}\end{pmatrix}\begin{pmatrix}\chi_1'\\ \chi_2'\\ \chi_3'\end{pmatrix}_L . \tag{8}$$

A recalculation of Eq. (6) with this current would seem to imply no *CP* violation. However, all we are doing is simply hiding the phase in the Majorana condition of the neutrino fields. The *CP*-violating phases will appear in the neutrino propagators,[8] and the physical consequences are the same as before. Notice that now we have

$$\begin{aligned}\chi_1'^c&=\chi_1' ,\\ \chi_2'^c&=e^{-2i\epsilon}\chi_2' ,\\ \chi_3'^c&=e^{-2i\eta}\chi_3' .\end{aligned} \tag{9}$$

If ξ or η equal either 0 or $\pi/2$ there will not be any *CP* violation in double-beta decay. In those cases χ_2' and χ_3' will have *CP* eigenvalues which are equal to or opposite from that of χ_1'. Then the double-beta-decay amplitude is proportional to

$$\overline{u}_e\gamma^\alpha(1-\gamma_5)(m_1C_1^2+\epsilon_2m_2S_1^2C_3^2+\epsilon_3m_3S_1^2S_3^2)(1-\gamma_5)\gamma^\beta v_e^c , \tag{10}$$

where ϵ_2 is the relative *CP* eigenvalue between χ_1' and χ_2', and ϵ_3 that between χ_1' and χ_3'. It is clear that with the right combination of mixing angles and masses and *CP* eigenvalues there can be cancellations between the contributions from different neutrinos. In the extreme case $m_1=m_2=m_3$, $\theta_1=\pi/4$, $\epsilon_3=\epsilon_2=-1$ the cancellation is complete.

It is instructive to analyze this last result in terms of pseudo-Dirac neutrinos. This is most easily done by redefining the parameters ξ and η in a way in which they do not include the phases of the mass eigenvalues. In that case, $\xi=\eta=0$ will correspond to no *CP* violation in double-beta decay, and the mass eigenvalues can be positive or negative, depending on the relative *CP* eigenvalue. In this form the mass matrix for the ν_i is

$$\begin{pmatrix} m & 0 & 0\\ 0 & -m & 0\\ 0 & 0 & -m\end{pmatrix} . \tag{11}$$

By defining

$$\nu_i'=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 0\\ 1 & 1 & 0\\ 0 & 0 & \sqrt{2}\end{pmatrix}\nu_i , \tag{12}$$

the mass matrix for the ν_i' now becomes

$$\begin{pmatrix} 0 & m & 0\\ m & 0 & 0\\ 0 & 0 & -m\end{pmatrix} . \tag{13}$$

We see that effectively the three Majorana neutrinos have been transformed into a pseudo-Dirac neutrino $\nu_D=\nu_{1L}'+\nu_{2R}^{c\prime}$ and a Majorana neutrino ν_M. The weak eigenstates are (for $\delta=0$)

$$\begin{aligned}\nu_{e_L}&=\tfrac{1}{2}(1+C_3)\nu_{D_L}+\tfrac{1}{2}(1-C_3)\nu_{D_L}^c-\frac{S_3}{\sqrt{2}}\nu_M ,\\ \nu_{\mu_L}&=\tfrac{1}{2}(C_2-C_2C_3+\sqrt{2}S_2S_3)\nu_{D_L}+\tfrac{1}{2}(C_2+C_2C_3-\sqrt{2}S_2S_3)\nu_{D_L}^c+\frac{1}{\sqrt{2}}(C_2S_3+\sqrt{2}S_2C_3)\nu_M ,\\ \nu_{\tau_L}&=\tfrac{1}{2}(S_2-S_2C_3-\sqrt{2}C_2S_3)\nu_{D_L}+\tfrac{1}{2}(S_2+S_2C_3+\sqrt{2}C_2S_3)\nu_{D_L}^c+\frac{1}{\sqrt{2}}(S_2S_3-\sqrt{2}C_2C_3)\nu_M .\end{aligned} \tag{14}$$

The mass matrix conserves $l_1'-l_2'$, which is the lepton number of ν_D. However, it is clear from Eq. (14) that this lepton number is not conserved by the weak interaction. It is evident that the presence of, and the mixing with, a third neutrino induces a small Majorana mass which splits ν_D into two Majorana neutrinos. In the limit $\theta_2=\theta_3=0$, then we obtain $\nu_{e_L}=\nu_{D_L}$, $\nu_{\mu_L}=\nu_{D_L}^c$, $\nu_{\tau_L}=\nu_M$, and the "prototypical" Dirac mass matrix.

Other processes may involve different parts of the mixing matrix and different effects occur. For example, an interesting case is $\mu^-\to e^+$ conversion.[10] The amplitude is proportional to

$$\sum_j m_jU_{ij}^*U_{2j}^*=S_1C_1C_2m_1-S_1C_3(C_1C_2C_3-S_2S_3e^{-i\delta})e^{-2i\xi}m_2-S_1S_3(C_1C_2S_3+S_2C_3e^{-i\delta})e^{-2i\eta}m_3 . \tag{15}$$

In this case, for $m_1=m_2=m_3$ the complete cancellation occurs for all neutrinos having the same *CP* eigenvalues ($\xi=\eta=0$) regardless of the values of the mixing angles.

Note that, in contrast with the case of two generations, the cancellations described above do not require complete *CP* conservation. If the parameters ξ and η have the values needed for cancellation, *CP* violation can still occur via the phase δ. In that sense, the situation would be very similar to that with three Dirac neutrinos, where the two processes discussed above could not occur and *CP* violation could take place with one *CP*-violating phase.

The possible cancellation in double-beta decay due to the interference between neutrinos of different *CP* eigenvalues was discussed by Wolfenstein[1] and others. As a possible mechanism for obtaining Majorana masses he mentioned a model by Zee[11] in which the original mass matrix has no diagonal elements. Thus, after diagonalizing with a unitary transformation the sum of the eigenvalues must vanish, guaranteeing that some of the Majorana fields must have opposite *CP* eigenvalues. It is interesting to note that, as discussed here, it is not necessary for the sum of the mass eigenvalues to vanish in order to have complete cancellation with three generations.

Finally, it should be noted that realistic models[6] based on SU(5), with the appropriate discrete symmetry on Higgs-boson and fermion couplings, can lead to mass matrices such as those discussed above.

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